

# Thermodynamical Interpretation of the Interacting Holographic Dark Energy Model in a non-flat Universe

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## Abstract

Motivated by the recent work of Wang, Lin, Pavon, and Abdalla [1], we generalize their work to the non-flat case. In particular, we provide a thermodynamical interpretation for the holographic dark energy model in a non-flat universe. For this case, the characteristic length is no more the radius of the event horizon ( $R_E$ ) but the event horizon radius as measured from the sphere of the horizon ( $L$ ). Furthermore, when interaction between the dark components of the holographic dark energy model in the non-flat universe is present its thermodynamical interpretation changes by a stable thermal fluctuation. A relation between the interaction term of the dark components and this thermal fluctuation is obtained. In the limiting case of a flat universe, i.e.  $k = 0$ , all results given in [1] are obtained.

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# 1 Introduction

Recent observations from type Ia supernovae [2] associated with Large Scale Structure [3] and Cosmic Microwave Background anisotropies [4] have provided main evidence for the cosmic acceleration. The combined analysis of cosmological observations suggests that the universe consists of about 70% dark energy, 30% dust matter (cold dark matter plus baryons), and negligible radiation. Although the nature and origin of dark energy are unknown, we still can propose some candidates to describe it, namely since we do not know where this dark energy comes from, and how to compute it from the first principles, we search for phenomenological models. The astronomical observations will then select one of these models. The most obvious theoretical candidate of dark energy is the cosmological constant  $\lambda$  (or vacuum energy) [5, 6] which has the equation of state parameter  $w = -1$ . However, as it is well known, there are two difficulties that arise from the cosmological constant scenario, namely the two famous cosmological constant problems — the “fine-tuning” problem and the “cosmic coincidence” problem [7]. An alternative proposal for dark energy is the dynamical dark energy scenario. This dynamical proposal is often realized by some scalar field mechanism which suggests that the specific energy form with negative pressure is provided by a scalar field evolving down a proper potential. So far, a plethora of scalar-field dark energy models have been studied, including quintessence [8], K-essence [9], tachyon [10], phantom [11], ghost condensate [12] and quintom [13], and so forth. It should be noted that the mainstream viewpoint regards the scalar-field dark energy models as an effective description of an underlying theory of dark energy. In addition, other proposals on dark energy include interacting dark energy models [14], braneworld models [15], Chaplygin gas models [16], and many others.

Currently, an interesting attempt for probing the nature of dark energy within the framework of quantum gravity (and thus compute it from first principles) is the so-called “Holographic Dark Energy” (HDE) proposal [17, 18, 19, 20]. It is well known that the holographic principle is an important result of the recent researches for exploring the quantum gravity (or string theory) [21]. The HDE model has been tested and constrained by various astronomical observations [22, 23] as well as by the Anthropic Principle [24]. Furthermore, the HDE model has been extended to include the spatial curvature contribution, i.e. the HDE model in non-flat space [25]. For other extensive studies, see e.g. [26].

It is known that the coincidence or, “why now” problem is easily solved in some models of HDE based on the fundamental assumption that matter and holographic dark energy do not conserve separately [27, 28]. In fact a suitable evolution of the Universe is obtained when, in addition to the holographic dark energy, an interaction (decay of dark energy to matter) is assumed.

Since we know neither the nature of dark energy nor the nature of dark matter, a microphysical interaction model is not available either. However, pressureless dark matter in interaction with holographic dark energy is more than just another model to describe an accelerated expansion of the universe. It provides a unifying view of different models which are viewed as different realizations of the Interacting HDE Model at the perturbative level [29]. Since the discovery of black hole thermodynamics in 1970, physicists have speculated on the thermodynamics of cosmological models in an accelerated expanding universe [30]. Related to the present work, for time-independent and time-dependent equations of state (EoS), the first and second laws of thermodynamics in a flat universe were investigated in [31]. In particular, for the case of a constant EoS, the first law of

thermodynamics is valid for the apparent horizon (Hubble horizon) but it does not hold for the event horizon when viewed as system's IR cut-off. When the EoS is assumed to be time-dependent, using a holographic model of dark energy in flat space, the same result is obtained: the event horizon, in contrast to the apparent horizon, does not satisfy the first law. Additionally, while the event horizon does not respect the second law of thermodynamics, it holds for the universe enclosed by the apparent horizon.

In the present paper we extend the work by Wang, Lin, Pavon, and Abdalla [1] to the interacting HDE model of dark energy in a non-flat universe, we study the thermodynamical interpretation of the interacting holographic dark energy model for a universe enveloped by the event horizon measured from the sphere of the horizon named  $L$ . The remainder of the paper is as follows. In Section 2 we generalize the thermodynamical picture of the non-interacting HDE model in a non-flat universe. In Section 3, we extend the thermodynamical picture in the case where there is an interaction term between the dark components of the HDE model. An expression for the interaction term in terms of a thermal fluctuation is given. In the limiting case of flat universe, we obtain the results derived in [1]. Finally, Section 4 is devoted to concluding remarks.

## 2 Thermodynamical Picture of the non-Interacting HDE model

In this section we consider the HDE model when there is no interaction between the holographic energy density  $\rho_X$  and the Cold Dark Matter (CDM)  $\rho_m$  with  $w_m = 0$ . In addition, non-dark components have been considered negligible and thus are not included. The third Friedmann equation describes the time evolution of the energy densities of the dark components. These equations are actually the continuity equations for the dark energy and CDM

$$\dot{\rho}_X + 3H(1 + w_X^0)\rho_X = 0, \quad (1)$$

$$\dot{\rho}_m + 3H\rho_m = 0 \quad (2)$$

where the quantity  $H = \dot{a}/a$  is the Hubble parameter and the superscript above the equation of state parameter,  $w_X$ , denotes that there is no interaction between the dark components. The non-interacting HDE model will be accommodated in the non-flat Friedmann-Robertson-Walker universe which is described by the line element

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right) \quad (3)$$

where  $a = a(t)$  is the scale factor of the non-flat Friedmann-Robertson-Walker universe and  $k$  denotes the curvature of space with  $k = 0, 1, -1$  for flat, closed and open universe, respectively. A closed universe with a small positive curvature ( $\Omega_k \sim 0.01$ ) is compatible with observations [32, 33]. Thus, in order to connect the curvature of the universe to the energy density, we employ the first Friedmann equation given by

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_p^2}[\rho_X + \rho_m] \quad (4)$$

where  $c$  is a positive constant in the HDE model and  $M_p$  is the reduced Planck mass. We also define the dimensionless density parameters

$$\Omega_m = \frac{\rho_m}{\rho_{cr}} = \frac{\rho_m}{3M_p^2 H^2} , \quad \Omega_X = \frac{\rho_X}{\rho_{cr}} = \frac{\rho_X}{3M_p^2 H^2} , \quad \Omega_k = \frac{k}{a^2 H^2} . \quad (5)$$

Therefore, we can rewrite the first Friedmann equation as

$$\Omega_m + \Omega_X - \Omega_k = 1 . \quad (6)$$

For completeness, we give the deceleration parameter

$$q = -\frac{\ddot{a}}{H^2 a} = -\left(\frac{\dot{H}}{H^2} + 1\right) \quad (7)$$

which combined with the Hubble parameter and the dimensionless density parameters form a set of useful parameters for the description of the astrophysical observations. It should be stressed that in the non-flat universe the characteristic length which plays the role of the IR-cutoff is the radius  $L$  of the event horizon measured on the sphere of the horizon and not the radius  $R_h$  measured on the radial direction. Therefore, the holographic dark energy density is given as

$$\rho_X = \frac{3c^2 M_p^2}{L^2} . \quad (8)$$

The radius  $L$  is given by

$$L = ar(t) \quad (9)$$

where the function  $r(t)$  is defined through the equation

$$\int_0^{r(t)} \frac{dr}{\sqrt{1 - kr^2}} = \frac{R_h}{a} . \quad (10)$$

Solving for the general case of non-flat universe the above equation, the function  $r(t)$  is given as

$$r(t) = \frac{1}{\sqrt{k}} \sin y \quad (11)$$

where

$$y = \frac{\sqrt{k} R_h}{a} . \quad (12)$$

Substituting equation (8) in the expression for the dimensionless density parameter of the holographic dark energy as given by equation (5), one gets

$$HL = \frac{c}{\sqrt{\Omega_X^0}} \quad (13)$$

and thus

$$\dot{L} = HL + ar(t) = \frac{c}{\sqrt{\Omega_X^0}} - \cos y . \quad (14)$$

Differentiating the holographic dark energy density as given by equation (8) and using equations (13) and (14), one gets

$$\dot{\rho}_X = -2H \left( 1 - \frac{\sqrt{\Omega_X^0}}{c} \cos y \right) \rho_X \quad (15)$$

and thus the conservation equation for the holographic dark energy (1) yields

$$1 + 3\omega_X^0 = -2 \frac{\sqrt{\Omega_X^0}}{c} \cos y . \quad (16)$$

Following [1] (see also [34]), the non-interacting HDE model in the non-flat universe as described above is thermodynamically interpreted as a state in thermodynamical equilibrium. According to the generalization of the black hole thermodynamics to the thermodynamics of cosmological models, we have taken the temperature of the event horizon to be  $T_L = (1/2\pi L)$  which is actually the only temperature to handle in the system. If the fluid temperature of the cosmological model is set equal to the horizon temperature ( $T_L$ ), then the system will be in equilibrium. Another possibility [35] is that the fluid temperature is proportional to the horizon temperature, i.e. for the fluid enveloped by the apparent horizon  $T = cH/2\pi$  [36]. In general, the systems must interact for some length of time before they can attain thermal equilibrium. In the case at hand, the interaction certainly exists as any variation in the energy density and/or pressure of the fluid will automatically induce a modification of the horizon radius via Einstein's equations. Moreover, if  $T \neq T_L$ , then energy would spontaneously flow between the horizon and the fluid (or viceversa), something at variance with the FRW geometry [37]. Thus, when we consider the thermal equilibrium state of the universe, the temperature of the universe is associated with the horizon temperature. In this picture the equilibrium entropy of the holographic dark energy is connected with its energy and pressure through the first thermodynamical law

$$TdS_X = dE_X + p_X dV \quad (17)$$

where the volume is given as

$$V = \frac{4\pi}{3} L^3 , \quad (18)$$

the energy of the holographic dark energy is defined as

$$E_X = \rho_X V = 4\pi c^2 M_p^2 L \quad (19)$$

and the temperature of the event horizon is given as

$$T = \frac{1}{2\pi L^0} . \quad (20)$$

Substituting the aforesaid expressions for the volume, energy, and temperature in equation (17) for the case of the non-interacting HDE model, one obtains

$$dS_X^{(0)} = 8\pi^2 c^2 M_p^2 (1 + 3\omega_X^0) L^0 dL^0 \quad (21)$$

and implementing equation (16) the above-mentioned equation takes the form

$$dS_X^{(0)} = -16\pi^2 c M_p^2 \sqrt{\Omega_X^0} \cos y L^0 dL^0 \quad (22)$$

where the superscript (0) denotes that in this thermodynamical picture our universe is in a thermodynamical stable equilibrium.

In the case of flat universe, i.e.  $k = 0$ , we obtain

$$dS_X^{(0)} = -16\pi^2 c M_p^2 \sqrt{\Omega_X^0} L^0 dL^0 \quad (23)$$

which is exactly the result derived in [1] when one replaces  $L^0$  with the future event horizon  $R_E^0$ .

### 3 Thermodynamical Picture of the Interacting HDE model

In this section we consider the HDE model when there is interaction between the holographic energy density  $\rho_X$  and the Cold Dark Matter (CDM)  $\rho_m$ . The corresponding continuity equations are now written as

$$\dot{\rho}_X + 3H(1 + w_X)\rho_X = -Q, \quad (24)$$

$$\dot{\rho}_m + 3H\rho_m = Q \quad (25)$$

where the quantity  $Q$  expresses the interaction between the dark components. The interaction term  $Q$  should be positive, i.e.  $Q > 0$ , which means that there is an energy transfer from the dark energy to dark matter. The positivity of the interaction term ensures that the second law of thermodynamics is fulfilled [34]. At this point, it should be stressed that the continuity equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor  $H$ ) multiplied with the energy density. Therefore, the interaction term could be in any of the following forms: (i)  $Q \propto H\rho_X$  [38, 34], (ii)  $Q \propto H\rho_m$  [39], or (iii)  $Q \propto H(\rho_X + \rho_m)$  [40]. The freedom of choosing the specific form of the interaction term  $Q$  stems from our incognizance of the origin and nature of dark energy as well as dark matter. Moreover, a microphysical model describing the interaction between the dark components of the universe is not available nowadays.

The interacting HDE model will again be accommodated in the non-flat Friedmann-Robertson-Walker universe described by the line element (3). Our analysis here will give same results with those in the non-interacting case concerning the first Friedmann equation, dimensionless density parameters, and the characteristic length as well as equations related to them (see equations (4) - (15)). However, due to the existence of interaction between the dark components of the holographic dark energy model which changed the conservation equations, equation (16) derived for the non-interacting HDE model has to be changed accordingly. Thus, by substituting equation (15) in the conservation equation (24) for the dark energy component one obtains

$$1 + 3\omega_X = -2\frac{\sqrt{\Omega_X}}{c} \cos y - \frac{Q}{3H^3 M_p^2 \Omega_X} . \quad (26)$$

Comparing equation (26) with equation (16), it is easily seen that the presence of the interaction term  $Q$  has provoked a change in the equation of state parameter and consequently in the dimensionless density parameter of the dark energy component and thus now there

is no subscript above the aforesaid quantities to denote the absence of interaction. According to [1], the interacting HDE model in the non-flat universe as described above is not anymore thermodynamically interpreted as a state in thermodynamical equilibrium. In this picture the effect of interaction between the dark components of the HDE model is thermodynamically interpreted as a small fluctuation around the thermal equilibrium. Therefore, the entropy of the interacting holographic dark energy is connected with its energy and pressure through the first thermodynamical law

$$TdS_X = dE_X + p_X dV \quad (27)$$

where now the entropy has been assigned an extra logarithmic correction [41]

$$S_X = S_X^{(0)} + S_X^{(1)} \quad (28)$$

where

$$S_X^{(1)} = -\frac{1}{2} \ln(CT^2) \quad (29)$$

and  $C$  is the heat capacity defined by

$$C = T \frac{\partial S_X^{(0)}}{\partial T} \quad (30)$$

and using equations (21), (20), and (16) is given as

$$C = -8\pi^2 c^2 M_p^2 (L^0)^2 (1 + 3\omega_X^0) \quad (31)$$

$$= 16\pi^2 c M_p^2 (L^0)^2 \sqrt{\Omega_X^0} \cos y . \quad (32)$$

Substituting the expressions for the volume, energy, and temperature (it is noteworthy that these quantities depend now on  $L$  and not on  $L^0$  since there is interaction among the dark components) in equation (27) for the case of the interacting HDE model, one obtains

$$dS_X = 8\pi^2 c^2 M_p^2 (1 + 3\omega_X) L dL \quad (33)$$

and thus one gets

$$1 + 3\omega_X = \frac{1}{8\pi^2 c^2 M_p^2 L} \frac{dS_X}{dL} \quad (34)$$

$$= \frac{1}{8\pi^2 c^2 M_p^2 L} \left[ \frac{dS_X^{(0)}}{dL} + \frac{dS_X^{(1)}}{dL} \right] \quad (35)$$

$$= -2 \left( \frac{\sqrt{\Omega_X^0}}{c} \cos y \right) \frac{L^0}{L} \frac{dL^0}{dL} + \frac{1}{8\pi^2 c^2 M_p^2 L} \frac{dS_X^{(1)}}{dL} \quad (36)$$

where the last term concerning the logarithmic correction can be computed using expressions (29) and (32)

$$\frac{dS_X^{(1)}}{dL} = - \frac{H}{\left( \frac{c}{\sqrt{\Omega_X^0}} - \cos y \right)} \left[ \frac{(\Omega_X^0)'}{4\Omega_X^0} + y \tan y \right] \quad (37)$$

with the prime ( ' ) to denote differentiation with respect to  $\ln a$ .

Therefore, by equating the expressions (26) and (36) for the equation of state parameter of the holographic dark energy evaluated on cosmological and thermodynamical grounds respectively, one gets an expression for the interaction term

$$\frac{Q}{9H^3 M_p^2} = \frac{\Omega_X}{3} \left[ -\frac{2\sqrt{\Omega_X}}{c} \cos y + \left( \frac{2\sqrt{\Omega_X}}{c} \cos y \right) \frac{L^0}{L} \frac{dL^0}{dL} \right] - \frac{1}{8\pi^2 c^2 M_p^2 L} \frac{\Omega_X}{3} \frac{dS_X^{(1)}}{dL} . \quad (38)$$

It is noteworthy that in the limiting case of flat universe, i.e.  $k = 0$ , we obtain exactly the result derived in [1] when one replaces  $L^0$  and  $L$  with  $R_E^0$  and  $R_E$ , respectively.

## 4 Conclusions

Understanding dark energy is one of the biggest challenges to the particle physics of this century. Studying the interaction between the dark energy and ordinary matter will open a possibility of detecting the dark energy. It should be pointed out that evidence was recently provided by the Abell Cluster A586 in support of the interaction between dark energy and dark matter [42]. However, despite the fact that numerous works have been performed till now, there are no strong observational bounds on the strength of this interaction [43]. This weakness to set stringent (observational or theoretical) constraints on the strength of the coupling between dark energy and dark matter stems from our unawareness of the nature and origin of dark components of the Universe. It is therefore more than obvious that further work is needed to this direction.

In 1973, Bekenstein [44] assumed that there is a relation between the area of the event horizon of a black hole and the thermodynamics of a black hole, so that the area of the event horizon of the black hole is a measure of the black hole entropy. Along this line of thought, it was argued in [45] that the gravitational Einstein equations can be derived through a thermodynamical argument using the relation between area and entropy as input. Following [45, 46], Danielsson [47] has been able to obtain the Friedmann equations, by applying the relation  $\delta Q = TdS$  to a cosmological horizon and calculate the heat flow through the horizon of an expanding universe in an acceleration phase. This idea has been generalized to horizons of cosmological models, so that each horizon corresponds to an entropy. Therefore, the second law of thermodynamics was modified in a way that in its generalized form, the sum of all time derivatives of entropies related to horizons plus the time derivative of normal entropy must be positive i.e. the sum of entropies must be an increasing function of time.

In the present paper, we have provided a thermodynamical interpretation for the HDE model in a non-flat universe. We utilized the horizon's radius  $L$  measured from the sphere of the horizon as the system's IR cut-off. We investigated the thermodynamical picture of the interacting HDE model for a non-flat universe enveloped by this horizon. The non-interacting HDE model in a non-flat universe was thermodynamically interpreted as a thermal equilibrium state. When an interaction between the dark components of the HDE model in the non-flat universe was introduced the thermodynamical interpretation of the HDE model changed. The thermal equilibrium state was perturbed by a stable thermal fluctuation which was now the thermodynamical interpretation of the interaction. Finally, we have derived an expression that connects this interaction term of the dark components of the interacting HDE model in a non-flat universe with the aforesaid thermal fluctuation.



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